

# Pion form factors

Mikhail Gorchtein,<sup>1</sup> Peng Guo,<sup>2</sup> and Adam P. Szczepaniak<sup>2,1</sup>

<sup>1</sup>*Center of Exploration of Energy and Matter, Indiana University, Bloomington, IN 47408*

<sup>2</sup>*Physics Department, Indiana University, Bloomington, IN 47405*

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We consider the electromagnetic and transition pion form factors. Using dispersion relations we simultaneously describe both the hadronic, time-like region and the asymptotic region of large energy-momentum transfer. For the latter we propose a novel mechanism of Regge fermion exchange. We find that hadronic contributions dominate form factors at all currently available energies.

Photons interact with quarks, the charged constituents of hadrons and the resulting electromagnetic form factors probe the quark energy-momentum distribution in hadrons. In this letter we examine the charged pion electromagnetic form factor  $F_{2\pi}(s)$ , which is defined by the matrix element  $\langle \pi^+(p')\pi^-(p)|J_\mu|0\rangle = e(p' - p)_\mu F_{2\pi}(s)$ , and the transition form factor between the neutral pion and a real photon,  $F_{\pi\gamma}(s)$  determined by  $\langle \pi^0(p')\gamma(\lambda, p)|J_\mu|0\rangle = ie^2/4\pi^2 f_\pi \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu}(\lambda) p'^\alpha p^\beta F_{\pi\gamma}(s)$ . Above,  $J_\mu$  is the electromagnetic current,  $s = (p' + p)^2$  is the four-momentum transfer squared and  $f_\pi = 92.4$  MeV is the pion decay constant. Current conservation implies  $F_{2\pi}(0) = 1$  and, in the chiral limit, axial anomaly determination of the  $\pi^0 \rightarrow 2\gamma$  decay leads to the expectation  $F_{\pi\gamma}(0) \approx 1$ . Because at short distances quark/gluon interactions are asymptotically free, it has been postulated that at high energy or momentum transfer,  $|s| \gg \mu^2$ , both form factors measure hard scattering of the photon with a small number of the QCD constituents [1–3]. One would then expect  $\mu^2 \sim O(1 \text{ GeV}^2)$ , which is the typical hadronic scale, however, given the current status of the data it seems that  $\mu^2$  could be as large as  $O(10\text{--}100 \text{ GeV}^2)$  [4, 5]. This implies that an alternative description of the underlying dynamics might be in order and the subject of applicability of pQCD to exclusive reactions has in fact a long history [6]. Perturbative QCD (pQCD) analysis of the form factor asymptotics assumes specific properties of certain non-perturbative quantities, *i.e.* the parton momentum distribution amplitudes in the low-momentum, “wee” region. If these had different behavior from what is assumed in the pQCD analysis the arguments leading to dominance of leading twist perturbative scattering would break down [7]. Such pion distribution amplitudes were considered recently in [8, 9], however, the authors used perturbative evolution to soften the wee region and use pQCD formulae. The available data on the pion electromagnetic form factor ranges up to  $|s| \lesssim 10 \text{ GeV}$  [10] and is approximately a factor of three above the asymptotic prediction [11]. Even more spectacular discrepancy is observed in the transition form factor recently measured by BaBar [12]. For momentum transfers as large as  $-s \approx 40 \text{ GeV}^2$  the measurement disagrees with the asymptotic prediction not only in normalization but also in the overall  $s$ -dependence. While pQCD predicts  $sF_{\pi\gamma}(s) \rightarrow 2f_\pi$  as  $|s| \rightarrow \infty$  [3], the data suggest

that the magnitude of  $-sF_{\pi\gamma}(s)$  grows with  $|s|$ .

Crossing symmetry implies that form factors in the space-like ( $s < 0$ ) and time-like ( $s > 0$ ) region are boundary values of an analytical function defined in the complex- $s$  plane with a unitarity cut running over the positive  $s$ -axis and starting at the two pion production threshold branch point  $s_{th} = 4m_\pi^2$ . In the time-like region the electromagnetic (transition) form factor describes the amplitude for production of a spin-1,  $\pi^+\pi^-$  pair ( $\pi^0\gamma$ ) in the external electromagnetic field of the virtual photon. In the space-like region the form factors are usually interpreted in terms of parton three-momentum distribution in a hadron (and/or photon). Analyticity demands these apparently distinct physical pictures to be smoothly connected. The dominant feature of the spin-1,  $\pi^+\pi^-$  state is the isovector  $\rho(770)$  resonance, which also dominates the electromagnetic form factor. There is no time-like data available for the transition form factor, however, also in this case one expects to see the  $\rho$ , and the isoscalar,  $\omega(782)$  resonance. The analytical continuation to the space-region implies that for  $-s \lesssim 1 \text{ GeV}^2$ , *i.e.* in the hadronic range, the quark wave function is dual to the vector-meson exchange in the crossed channel.

In the following, we relate the space-like and time-like regions through dispersion relations, and focus on the dynamics in the asymptotic region,  $s \rightarrow +\infty$ . In view of the BaBar “anomaly” and the apparent failure of the pQCD description, we propose a novel description for the dominant mechanism that drives the asymptotic behavior of the form factors.

The discontinuity of  $F_{\pi\gamma}(s)$  across the unitary cut is given by

$$\text{Im}F_{\pi\gamma} = t_{2\pi, \pi\gamma}^* \rho_{2\pi} F_{2\pi} + t_{3\pi, \pi\gamma}^* \rho_{3\pi} F_{3\pi} + \sum_X t_{X, \pi\gamma}^* \rho_X F_X \quad (1)$$

and the sum runs over all possible intermediate states  $X \neq 2\pi, 3\pi$ . Here,  $t_{X, \pi\gamma}(F_X)$  represent the amplitudes for  $X \rightarrow \pi^0\gamma$  ( $\gamma^* \rightarrow X$ ), respectively and  $\rho_X$  is a product of the phase space and kinematical factors (*i.e.* for the  $2\pi$  intermediate state  $\rho_{2\pi}(s) = s(1 - s_{th}/s)^{3/2}/96\pi$ ). Provided  $\text{Im}F_{\pi\gamma}$  vanishes at  $s \rightarrow \infty$ , its real part can be reconstructed for any  $s$  from the unsubtracted dispersion relation

$$F_{\pi\gamma}(s) = \frac{1}{\pi} \int_{s_{th}} ds' \frac{\text{Im}F_{\pi\gamma}(s')}{s' - s}. \quad (2)$$

The two lowest mass intermediate states,  $X = 2\pi, 3\pi$  that are dominated by the  $\rho(770)$  and  $\omega(782)$  resonances, respectively, are expected to saturate the cut in the hadronic range  $s_{th} < s \lesssim 1\text{GeV}^2$ . The  $\omega(782)$  in the isoscalar  $3\pi$  channel is a narrow resonance with width to mass ratio,  $\Gamma_\omega/m_\omega \sim 10^{-2}$  and its contribution to  $F_{\pi\gamma}$  can be well approximated by a Breit-Wigner distribution,

$$F_{\pi\gamma}^{(3\pi)}(s) = \frac{c_{\pi\gamma}^{(3\pi)} m_\omega^2}{m_\omega^2 - s - im_\omega \Gamma_\omega(s)} \quad (3)$$

with  $c_{\pi\gamma}^{(3\pi)} = 4\pi^2 f_\pi g_{\omega\pi\gamma}/m_\omega g_\omega = 0.493$  obtained from  $\omega \rightarrow \pi\gamma$  and  $\omega \rightarrow e^+e^-$  decay widths yielding  $g_{\omega\pi\gamma} = 1.81$  and  $g_\omega = 17.1$ , respectively. The contribution from the  $2\pi$  intermediate state is dominated by the  $\rho(770)$  resonance, which determines both the  $t_{2\pi,\pi\gamma}$  scattering amplitude and the pion electromagnetic form factor,  $F_{2\pi}$  for  $s \lesssim 1\text{GeV}^2$ , and vector meson dominance (VMD), yields  $c_{\pi\gamma}^{(2\pi)} = 4\pi^2 f_\pi g_{\rho\pi\gamma}/m_\rho g_\rho = 0.613$  (with  $\rho \rightarrow \pi\gamma$  and  $\rho \rightarrow e^+e^-$  decay widths leading to  $g_{\rho\pi\gamma} = 0.647$  and  $g_\rho = 4.96$ ). At  $s = 0$  the sum of the two resonance contributions to  $F_{\pi\gamma}$  agree with the anomaly driven normalization to within 10-15% and the isovector contribution can be further improved using a unitary parametrization of [13, 14], which for  $F_{2\pi}$  and  $t_{2\pi,\pi\gamma}$  in Eq. (1) yields,

$$F_{2\pi}(s) = P(s)\Omega(s), \quad t_{2\pi,\pi\gamma}(s) = C^{-1}(s)\Omega(s), \quad (4)$$

where  $\Omega(s)$ , the Omnes-Muskhelishvili function [15] computed from the phase of the vector-isovector elastic  $\pi\pi$  scattering amplitude and satisfying the VMD relation,  $\Omega(s \sim m_\rho^2) \sim m_\rho^2/(m_\rho^2 - s - im_\rho \Gamma_\rho(s))$ . The polynomials  $P(s)$  and  $C(s)$  ( $P(s) = 1 + 0.1s/m_\rho^2$ ,  $C(s) = f_\pi^2[1 + 1.27s/m_\rho^2 + 1.38s^2/m_\rho^4 - 0.50s^3/m_\rho^6]$ ) are determined from fits to the electromagnetic form factor and the solution to the dispersion relation for the  $t_{2\pi,\pi\gamma}$  amplitude, respectively. At higher energies,  $s \gtrsim 1\text{GeV}^2$  the  $K\bar{K}$  inelastic channel and other multi-particle intermediate states are expected to contribute. Unfortunately, since no time-like data are available (unlike in the case of the electromagnetic form factor) one cannot unambiguously determine these contributions. A possible determination of the multi-particle hadronic states could be given in terms of quark/gluon intermediate states, much like in the derivation of QCD sum rules (*cf.* Ref. [16] for the case of the pion electromagnetic form factor).

Since the electromagnetic form factor  $F_X$  of a composite state decreases with energy-momentum transfer, asymptotically the *r.h.s* of Eq. (1) is dominated by the  $X = q\bar{q}$ , quark-antiquark intermediate state. Its form factor is  $F_{q\bar{q}} = 1$ , (in units of the quark charge) and the state contributes to  $ImF_{\pi\gamma}$  via the  $q\bar{q} \rightarrow \pi\gamma$ ,  $P$ -wave scattering amplitude,  $t_{q\bar{q},\pi\gamma}$  as illustrated by the last diagram in Fig.1a. The  $q\bar{q}$  contribution shown in Fig.1a may be compared to the one in Fig.1b, which represents the asymptotic contribution as predicted by pQCD. In the latter, the  $q\bar{q} \rightarrow \pi\gamma$  scattering amplitude, shown to the

right of the vertical cut line, is given by a free quark propagator exchanged between the final state pion and photon. In the kinematically relevant domain  $s \gg t \sim b^{-1}$  with  $t$  being the four momentum squared carried by the exchanged quark and  $b \approx \text{few GeV}^{-2}$  the typical slope of the product of residual coupling of the exchange quark ( $\beta_\pi, \beta_\gamma$ ), the amplitude  $t_{q\bar{q},\pi\gamma}$  is expected to have a Regge behavior [17]

$$t_{q\bar{q},\pi\gamma}(s, t) = \beta_\pi(t)\beta_\gamma(t)s^{\alpha_q(t)} \approx e^{bt}s^{\alpha_q} \quad (5)$$

The difference between the free, Fig.1b and the Regge

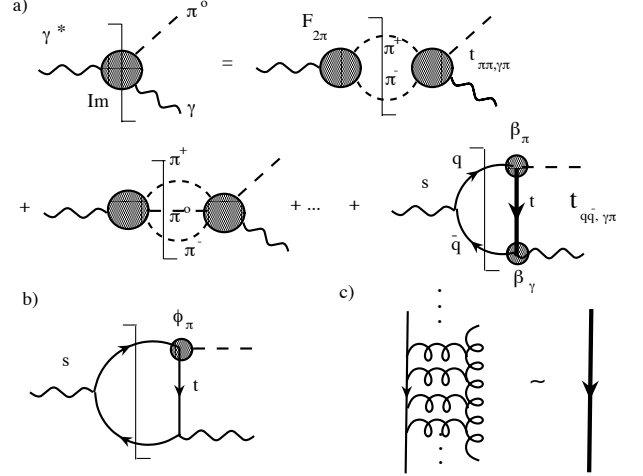


FIG. 1: Hadronic and asymptotic contributions to the  $\pi^0$  transition form factor.

propagator Fig.1a can originate from the sum of ladder gluons in the wee region (*cf.* Fig.1c). The quark Regge trajectory  $\alpha_q(t) \approx \alpha_q(0) + \alpha'_q t$  is not known, however, phenomenologically it can be related to the leading Regge exchange in  $\pi\pi$  scattering, *i.e.* the  $\rho$  or  $f_2$  exchange (that are nearly degenerate, *cf.* [18]),  $\alpha_\rho(t) \sim \alpha_{f_2}(t)$ ,

$$\alpha_q(t) \sim 0.5\alpha_\rho(t) + 0.5 \approx 0.75 + 0.45t. \quad (6)$$

It is worth noting that the dominance of quark-exchange (or, more precisely, quark *interchange*) mechanism has previously been observed in hard scattering processes with hadrons [19]. Hard scattering data are furthermore compatible with  $\alpha_q(t = -1\text{GeV}^2) \approx 0.3 - 0.4$  [20], which is consistent with Eq. (6). Detailed derivation of Eq. (6) *i.e.* relation between quark and meson Regge trajectories will be given in the forthcoming paper [21]. After projecting onto spin-1 partial wave, the energy dependence of the asymptotic,  $q\bar{q}$  contribution to  $ImF_{\pi\gamma}$  is therefore expected to behave as (modulo terms  $\sim O(\ln s)$ ),

$$ImF_{\gamma^*\pi\gamma}^{(q\bar{q})}(s) \rightarrow c_{2\pi}^{(q\bar{q})} s^{\alpha_q(0)-3/2}. \quad (7)$$

The important point is that with  $\alpha_q(0) = 1/2 + \epsilon$  (assuming  $\alpha'_q(0) > 0$ , it is consistent with the absence of a physical pole for a confined quark) Eq. (7) implies asymptotic increase of the energy weighted transition form factor,

$sF_{\pi\gamma}(s) \propto s^\epsilon$  in agreement with the BaBar measurement. Such an increase cannot be accounted for by the exchange of the free quark as it is the case for the leading twist pQCD model. Combining the  $\omega$  and the  $\rho$  resonance contributions of Eqs. (3),(4) with the asymptotic form of Eq. (7) and making the simplifying assumption that the first two contribute to  $ImF_{\pi\gamma}$  for  $s < 1 \text{ GeV}^2$  while the asymptotic part saturates  $ImF_{\pi\gamma}$  for  $s > \mu^2$ , we fit the available data using Eq. (2) with the single free parameter  $c_{\pi\gamma}^{(q\bar{q})}$  that determines the normalization of the asymptotic contribution. The result is shown in Fig.2. It is worth noting that even at largest values of  $-s$  the bare  $q\bar{q}$  production gives only about 50% of the form factor with the remaining half coming from the resonances.

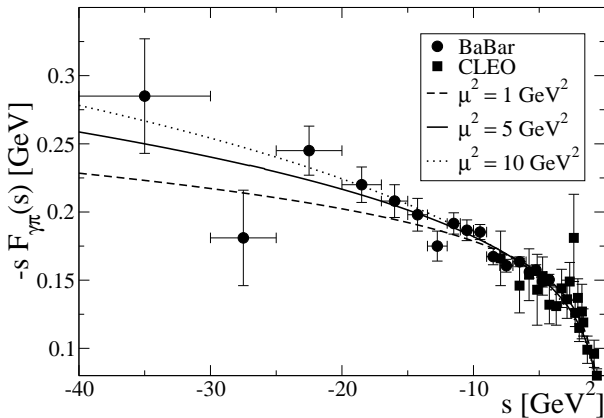


FIG. 2: Our results for  $|F_{\pi\gamma}(s)|$  in the space-like region for  $\mu^2 = 1 \text{ GeV}^2$  (dashed line),  $5 \text{ GeV}^2$  (solid line),  $10 \text{ GeV}^2$  (dotted line), in comparison with the experimental data from [12, 22].

In the case of the pion electromagnetic form factor, the discontinuity across the unitary cut is given by

$$ImF_{2\pi} = t_{2\pi,2\pi}^* \rho_{2\pi} F_{2\pi} + t_{K\bar{K},2\pi}^* \rho_{2K} F_K + \sum_X t_{X,2\pi}^* \rho_X F_X \quad (8)$$

where in the sum is over intermediate states ( $X \neq 2\pi, K\bar{K}$ ) in  $\gamma^* \rightarrow X \rightarrow 2\pi$  and where we separated the two channels  $X = 2\pi$  and  $X = K\bar{K}$  which phenomenologically are most significant in the hadronic domain [23]. Above the inelastic threshold,  $s > s_i$ , the unitarity relation now involves both  $ImF_{2\pi}$  and  $ReF_{2\pi}$  and can be solved algebraically. Assuming that the elastic amplitude,  $t_{2\pi,2\pi}$  asymptotically approaches the diffractive limit,  $t_{2\pi,2\pi} \rightarrow i/2\rho_{2\pi}$ , from Eq. (8) one finds

$$F_{2\pi}(s) \rightarrow 2i \sum_{X \neq 2\pi} t_{X,2\pi} \rho_X F_X^* \rightarrow 2it_{q\bar{q},2\pi} \propto is^{\alpha_q(0)-3/2}. \quad (9)$$

Except for the expected energy dependence *cf.* Eq. (5), we do not know separately the real and imaginary parts of  $t_{q\bar{q},2\pi}$ . Assuming, as in the case of the transition

form factor, that the real part of the discontinuity due to  $q\bar{q}$  state has the energy dependence given by the regigized quark exchange, we can compute  $F_{2\pi}$  using Eq. (8) and the Cauchy representation (the imaginary part of  $t_{q\bar{q},2\pi}^* \rho_X$  would then be given by the solution of an algebraic equation that follows from Eq. (8)). This yields,  $F_{2\pi}(s) = N(s)/D(s)$  with

$$N(s) = \sum_{X \neq 2\pi} \frac{1}{\pi} \int_{s_i} ds' \frac{D(s') Re[t_{X,2\pi}^*(s') \rho_X(s') F_X(s')]}{[1 - it_{2\pi,2\pi}^*(s') \rho_{2\pi}(s')](s' - s)}$$

$$D(s) = \exp\left(-\frac{s}{\pi} \int_{s_{th}} ds' \frac{\phi(s')}{(s' - s)s'}\right). \quad (10)$$

The phase  $\phi$  is obtained from the elastic amplitude,  $\tan \phi = Re t_{2\pi,2\pi} \rho_{2\pi} / (1 - Im t_{2\pi,2\pi} \rho_{2\pi})$ . As discussed earlier, the dominant feature of the pion electromagnetic form factor is the  $\rho(770)$  resonance. Close to the resonance peak there is also a contribution from the isospin-violating  $\omega \rightarrow 2\pi$  decay. Here we do not attempt to reproduce the details of the  $\rho - \omega$  interference region. The next relevant feature is the large variation in magnitude of  $|F_{2\pi}|$  at  $\sqrt{s} \sim 1.7 \text{ GeV}$  which is also seen in the elastic  $2\pi \rightarrow 2\pi$  amplitude and is attributed to the contribution from the inelastic  $\rho''(1700)$  resonance decaying to  $K\bar{K}$ . We thus approximate the sum over inelastic channels in Eq. (10) by the single  $K\bar{K}$  channel, and above  $s \geq \mu^2$  the residual  $q\bar{q}$  continuum with

$$Re t_{q\bar{q},2\pi}^* \rho_X = c_{2\pi}^{(q\bar{q})} s^{\alpha_q(0)-3/2}. \quad (11)$$

For the  $t_{2\pi,2\pi}$  and  $t_{K\bar{K},2\pi}$  amplitudes we use the parametrization from [26]. Even though the contribution to the dispersive integral from the high energy tails of  $t_{2\pi,2\pi}$  and  $t_{K\bar{K},2\pi}$  are suppressed by form factors *cf.* Eq. (8) we nevertheless extend the parametrization from [26] to higher energies by smoothly joining the resonance region to the spin-1 projected Regge limit of  $\pi\pi \rightarrow \pi\pi$  and  $K\bar{K} \rightarrow \pi\pi$  amplitudes. We parametrize the isovector kaon form factor  $F_K$  using Breit-Wigner distributions which include the  $\rho(770)$ ,  $\rho'(1400)$  and  $\rho''(1700)$  [24]. Finally we fit the available data on  $|F_{2\pi}(s)|^2$  (excluding the  $\rho - \omega$  interference region) with five parameters: the magnitude and phase of the  $\rho'$  and  $\rho''$  contributions to  $F_K$  and  $c_{2\pi}^{(q\bar{q})}$ —the magnitude of the  $q\bar{q}$  continuum, Eq. (11). In Fig. 3, we display our results for the electromagnetic pion form factor  $F_\pi$  in the range  $-40 \text{ GeV}^2 \leq s \leq 10 \text{ GeV}^2$ . We confront them with the available experimental data for the electromagnetic form factor for  $-10 \text{ GeV}^2 \leq s \leq 10 \text{ GeV}^2$  and the transition form factor for  $-40 \text{ GeV}^2 \leq s \leq -0.8 \text{ GeV}^2$  (both are normalized to 1 at  $s = 0$ ). First, we note that in the space-like region the data sets for the two form factors look identical (this is not expected to be the case for the time-like region since, for example the  $\omega(782)$  only contributes to the transition form factor). One can see that our model describes all the available data throughout the shown kinematics. This serves as an *a posteriori* evidence that this  $s$ -dependence is in both cases driven

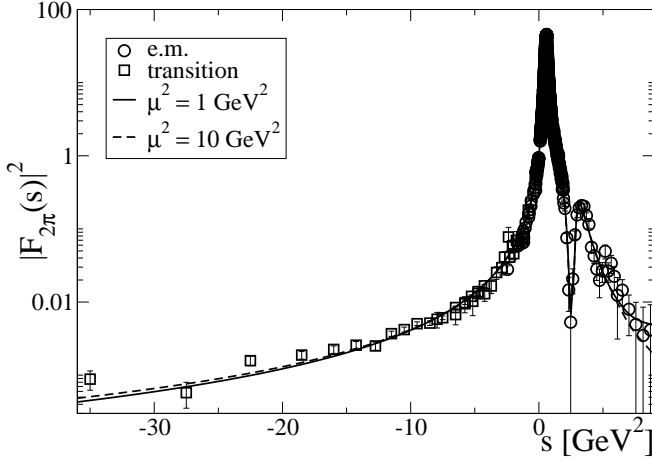


FIG. 3: Our results for the pion electromagnetic form factor for  $\mu^2 = 1 \text{ GeV}^2$  (solid line) and  $\mu^2 = 10 \text{ GeV}^2$  (dashed line) vs. experimental data on the time-like and space-like e.-m. form factor from [25] (solid circles).

by the same mechanism. In the case of the electromagnetic form factor, our result is a prediction for the  $s$ -dependence at large  $|s|$ , where no data exist so far. In particular, we predict that, as for the transition form

factor case,  $|s F_{2\pi(s)}|$  has to rise asymptotically roughly as  $s^{1/4}$ , unlike pQCD predictions that feature at most a logarithmic limit for that combination.

To summarize, we presented a calculation of the transition and electromagnetic form factors of the pion. We used dispersion relations to provide a unified description of the pion form factors in the time-like and space-like regions. In the hadronic energy range, we accounted for hadronic, resonance mechanisms in a fully unitarized manner. For asymptotic contributions, we proposed a new mechanism that features a reggeized quark exchange. We relate the parameters of such an exchange to  $\pi\pi$  scattering data and deduce that the quark-Regge intercept is approximately  $\alpha_q(0) \sim 3/4$ . Using this value as input for the asymptotic behavior of the pion form factors, we obtain for both  $sF(s) \propto s^{1/4}$ , in agreement with the recent BaBar data. We notice that when the transition form factor is renormalized so that  $F_{\pi\gamma}(0) = 1$  its dependence on  $s$  in the space-like region is consistent with that of the electromagnetic form factor, as shown by the open circles in Fig.3. We use the normalization of the Regge-behaved  $t_{q\bar{q},2\pi}$  and  $t_{q\bar{q},\pi\gamma}$  as the only free parameter, and are able to describe all available data on pion form factors.

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